PHY130

Review Chapters 13 and 14

Topic Summary

- **Hooke's Law**

$$F_s = -kx \qquad a = -\frac{k}{m}x$$

- **Elastic Potential Energy**

$$PE_s \equiv \frac{1}{2}kx^2 \qquad v = \pm\sqrt{\frac{k}{m}\left(A^2 + x^2\right)}$$

3. The force constant of a spring is 137 N/m. Find the magnitude of the force required to

a. compress the spring by 4.80 cm from its unstretched length and

Answer

b. stretch the spring by 7.36 cm from its unstretched length.

13.3 Assuming the spring obeys Hooke's law, the magnitude of the force required to displace the end a distance $|x|$ from the equilibrium position (by either compressing or stretching the spring) is $|F| = k|x|$, where k is the force constant of the spring.

(a) If $x = -4.80$ cm, the required force is $|F| = k|x| =$

$(137 \text{ N/m})(4.80 \times 10^{-2} \text{ m}) = \boxed{6.58 \text{ N}}$

(b) If $x = +7.36$ cm, the required force is $|F| = k|x| =$

$(137 \text{ N/m})(7.36 \times 10^{-2} \text{ m}) = \boxed{10.1 \text{ N}}$

25. A vertical spring stretches 3.9 cm when a 10.-g object is hung from it. The object is replaced with a block of mass 25 g that oscillates up and down in simple harmonic motion. Calculate the period of motion.

13.25 The spring constant is found from

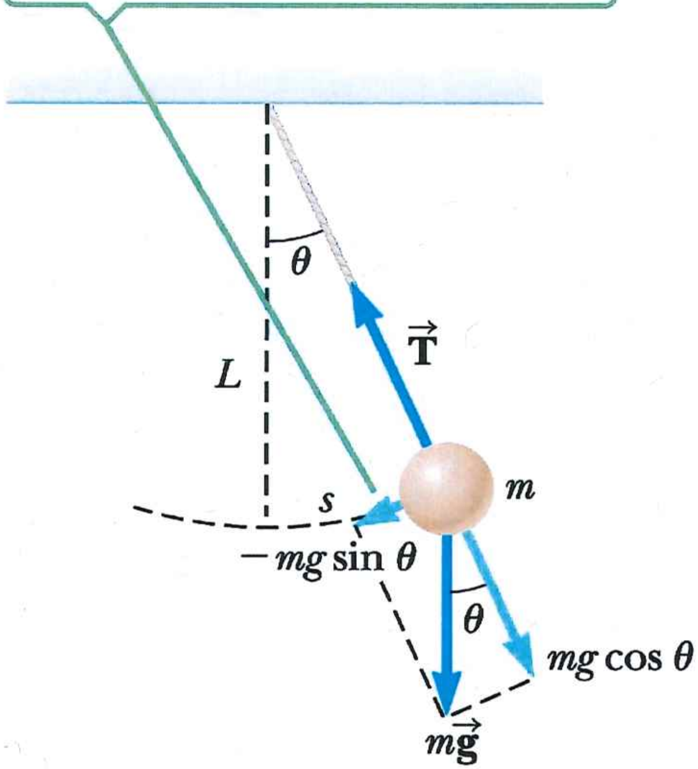
$$k = \frac{F_s}{x} = \frac{mg}{x} = \frac{(0.010\ \text{kg})(9.80\ \text{m/s}^2)}{3.9\times 10^{-2}\ \text{m}} = 2.5\ \text{N/m}$$

When the object attached to the spring has mass $m = 25$ g, the period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.025\ \text{kg}}{2.5\ \text{N/m}}} = \boxed{0.63\ \text{s}}$$

Motion of a Pendulum

The restoring force causing the pendulum to oscillate harmonically is the tangential component of the gravity force $-mg \sin\theta$.



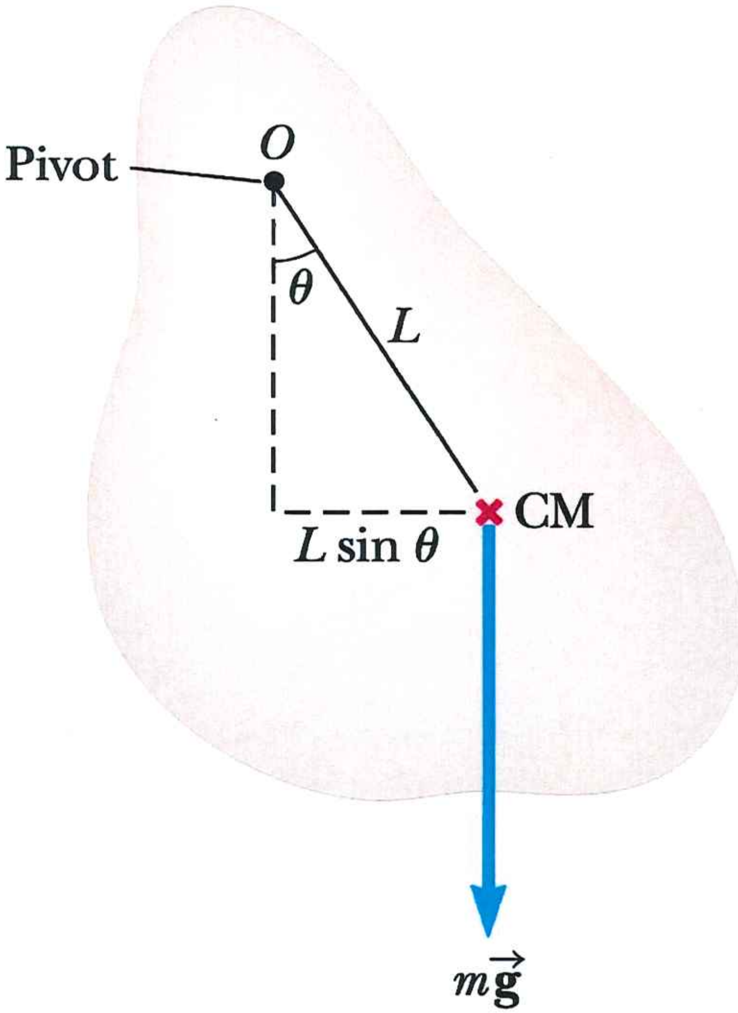
$$F_t = -mg\sin\theta$$

$$F_t = -mg\sin\left(\frac{s}{L}\right)$$

$$s = L\theta$$

$$F_t = -mg\sin\left(\frac{s}{L}\right)$$

The Physical Pendulum



$$T = 2\pi\sqrt{\frac{I}{mgL}}$$

simple pendulum:

$$I = mL^2$$

$$T = 2\pi\sqrt{\frac{mL^2}{mgL}} = 2\pi\sqrt{\frac{L}{g}}$$

34. A man enters a tall tower, needing to know its height. He notes that a long pendulum extends from the ceiling almost to the floor and that its period is 15.5 s.

a. How tall is the tower?

b. If this pendulum is taken to the Moon, where the free-fall acceleration is $1.67\ \mathrm{m/s^2}$, what is the period there?

13.34 **(a)** The height of the tower is almost the same as the length of the pendulum. From $T = 2\pi\sqrt{L/g}$, we obtain

$$L = \frac{gT^2}{4\pi^2} = \frac{(9.80\ \mathrm{m/s^2})(15.5\ \mathrm{s})^2}{4\pi^2} = \boxed{59.6\ \mathrm{m}}$$

(b) On the Moon, where $g = 1.67\ \mathrm{m/s^2}$, the period will be

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{59.6\ \mathrm{m}}{1.67\ \mathrm{m/s^2}}} = \boxed{37.5\ \mathrm{s}}$$

Ultrasound with f=4.8 MHz is used in medical imager. Find the wavelength in

i) Air where sound speed is 343 m/s
ii) In muscle tissues where sound speed is 1580 m/s

Answers:

$\lambda = V \cdot T = V/f$

i)

SOLVE Plugging in values:

Part (a): Wavelength in air is $\lambda = \frac{343 \text{ m/s}}{4.8 \times 10^6 \text{ Hz}} = 71\ \mu\text{m}.$

Part (b): Wavelength in muscle is $\lambda = \frac{1580 \text{ m/s}}{4.8 \times 10^6 \text{ Hz}} = 330\ \mu\text{m}.$

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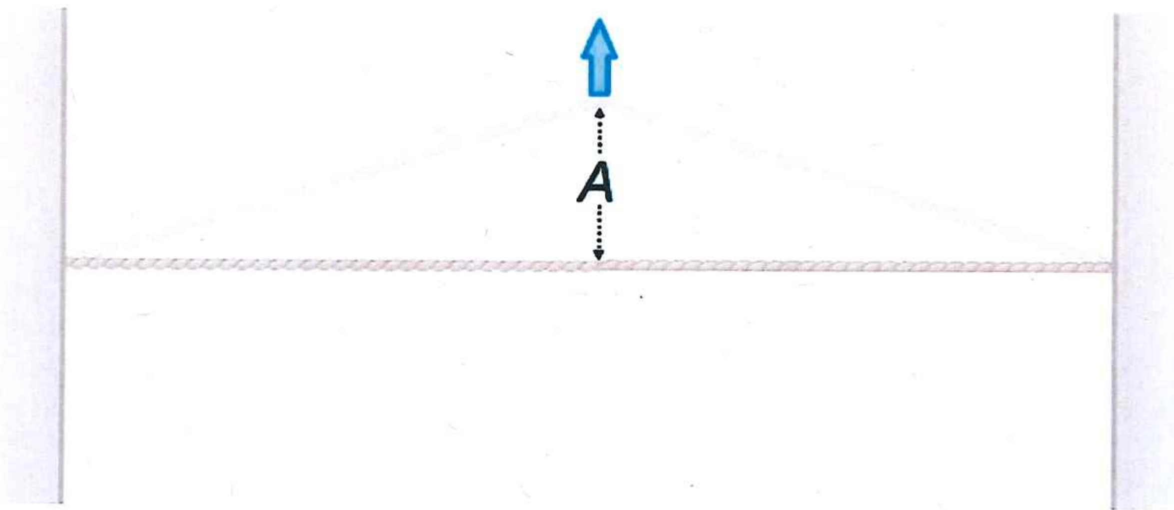
iii)

The Speed of Waves on Strings



$$v = \frac{\lambda}{T}$$

$$v = \sqrt{\frac{F}{\mu}}$$

53. Transverse waves with a speed of 50.0 m/s are to be produced on a stretched string. A 5.00-m length of string with a total mass of 0.060 0 kg is used.

a. What is the required tension in the string?

Answer

b. Calculate the wave speed in the string if the tension is 8.00 N.

13.53 (a) The mass per unit length is

$$\mu = \frac{m}{L} = \frac{0.060\ 0\ \text{kg}}{5.00\ \text{m}} = 1.20\times10^{-2}\ \text{kg/m}$$

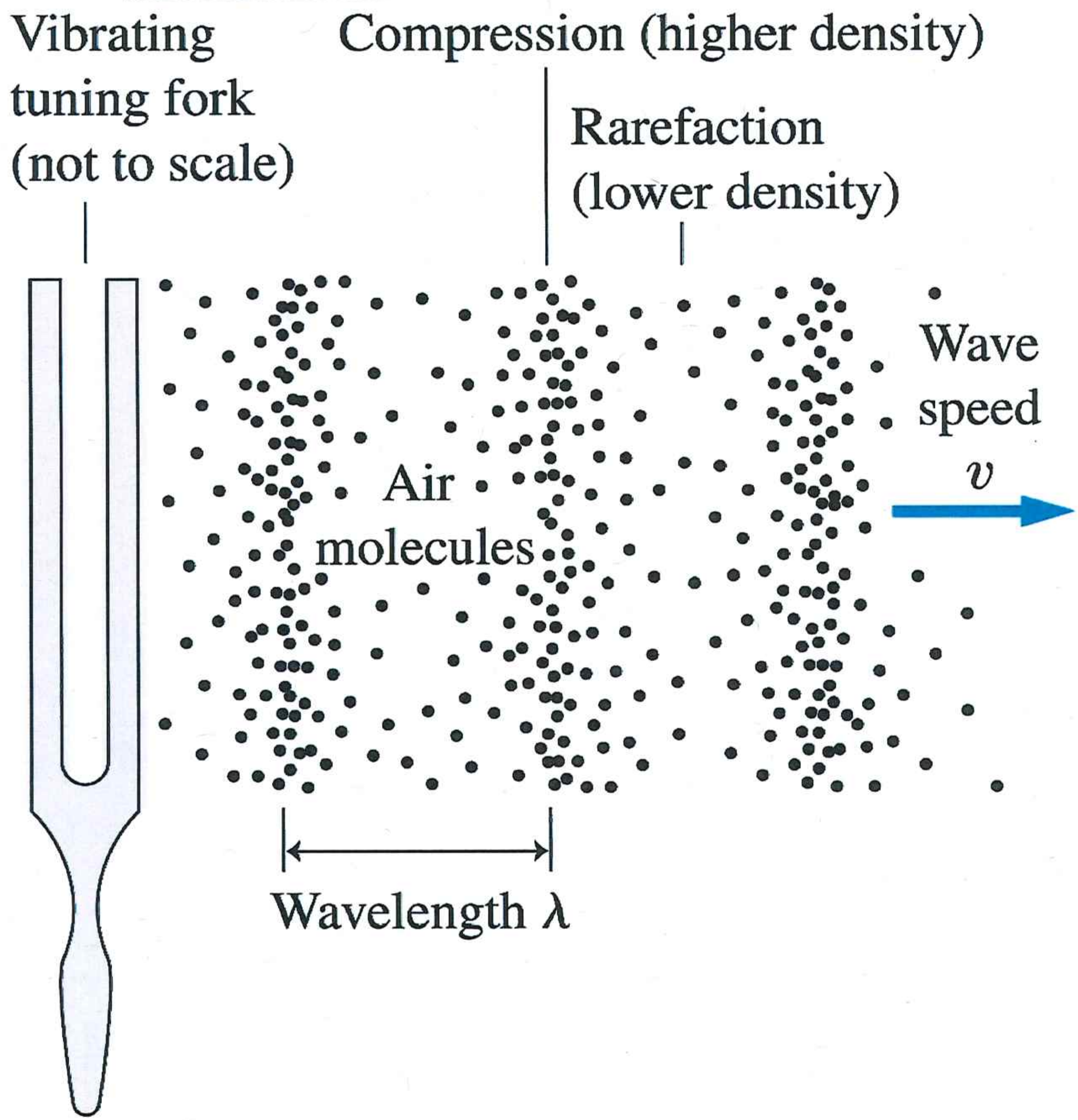
From $v = \sqrt{F/\mu}$, the required tension in the string is

$$F = v^2\mu = (50.0\ \text{m/s})^2(1.20\times10^{-2}\ \text{kg/m}) = \boxed{30.0\ \text{N}}$$

(b) $$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{8.00\ \text{N}}{1.20\times10^{-2}\ \text{kg/m}}} = \boxed{25.8\ \text{m/s}}$$

Sound waves

Figure 11.12



$f = 4.5$ MHz

$\lambda = V \cdot T = \frac{V}{f}$

$\lambda_{air} = ?$, $V_{tissue} = 1500 \frac{m}{s}$

$\lambda_T = ?$ $V_{air} = 343 \frac{m}{s}$

9. (a) Using $\lambda = v/f$, where v is the speed of sound in air and f is the frequency, we find

$$\lambda = \frac{343\,\text{m/s}}{4.5 \times 10^6\,\text{Hz}} = 7.62 \times 10^{-5}\ \text{m}\ .$$

(b) Now, $\lambda = v/f$, where v is the speed of sound in tissue. The frequency is the same for air and tissue. Thus $\lambda = (1500\,\text{m/s})/(4.5 \times 10^6\,\text{Hz}) = 3.33 \times 10^{-4}$ m.

The Speed of Sound

$$v = \sqrt{\frac{B}{\rho}} \qquad B \equiv -\frac{\Delta P}{\Delta V / V}$$

$$v = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}}$$

$$v = \sqrt{\frac{F}{\mu}} \; (\text{transverse wave on a string})$$

$$v = \sqrt{\frac{Y}{\rho}} \; (\text{longitudinal wave in a rod})$$

9. A hammer strikes one end of a thick steel rail of length 8.50 m. A microphone located at the opposite end of the rail detects two pulses of sound, one that travels through the air and a longitudinal wave that travels through the rail.

a. Which pulse reaches the microphone first?

Answer

b. Find the separation in time between the arrivals of the two pulses.

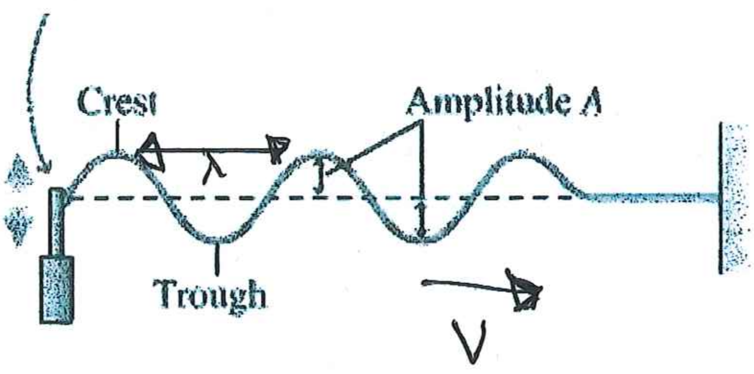
14.9 (a) Because the speed of sound in air is $v_{air} = 343$ m/s while its speed in the steel rail is

$v_{steel} = 5\,950$ m/s, the pulse traveling in the steel rail arrives first.

(b) The difference in times when the two pulses reach the microphone at the opposite end of the rail is

$$\Delta t = \frac{L}{v_{air}} - \frac{L}{v_{steel}} = (8.50\text{ m})\left(\frac{1}{343\text{ m/s}} - \frac{1}{5\,950\text{ m/s}}\right) = 2.34\times10^{-2}\text{ s} = \boxed{23.4\text{ ms}}$$

What is the speed of a wave with a wavelength of 15 cm and frequency of 2 kHz ?



$\lambda = v.T$

and $T = 1/f$

where $f = 2$ KHz $= 2000$ Hz

and $\lambda = 15$ cm $= 0.15$ m

then $\lambda = v.T = v/f$ and $v = \lambda.f = (0.15m).(2000\ Hz) = 300\ m/s$

$V = 300\frac{m}{s}$; $\lambda = 1.5\,m$; $f = ?$

1 Given values: frequency v and wavelength λ. We are asked to find frequency f. The fundamental relationship connecting these three values is:

$$v = \lambda f$$; $V = \frac{\lambda}{T}$; $T = \frac{1}{f}$

Solving for f yields:

$$f = \frac{v}{\lambda}$$

SOLVE Plugging in values yields:

$$f = \frac{360 \text{ m/s}}{1.5 \text{ m}} = 240 \text{ Hz}$$

$\lambda = 1.55\text{ m}$ fixed; v_2? | $f_1 = 0.365\text{ Hz}$
$f_2 = 2f_1$

We are given values for wavelength and frequency and asked to determine wave speed. We will employ the fundamental relationship: $v = \lambda f$

SOLVE Plugging in values: $V = \frac{\lambda}{T} = \lambda \cdot f$

Part (a): $v = 1.55\ \text{m} \times 0.365\ \text{Hz} = 0.566\ \text{m/s}$

Part (b): $v = 1.55\ \text{m} \times 0.730\ \text{Hz} = 1.13\ \text{m/s}$

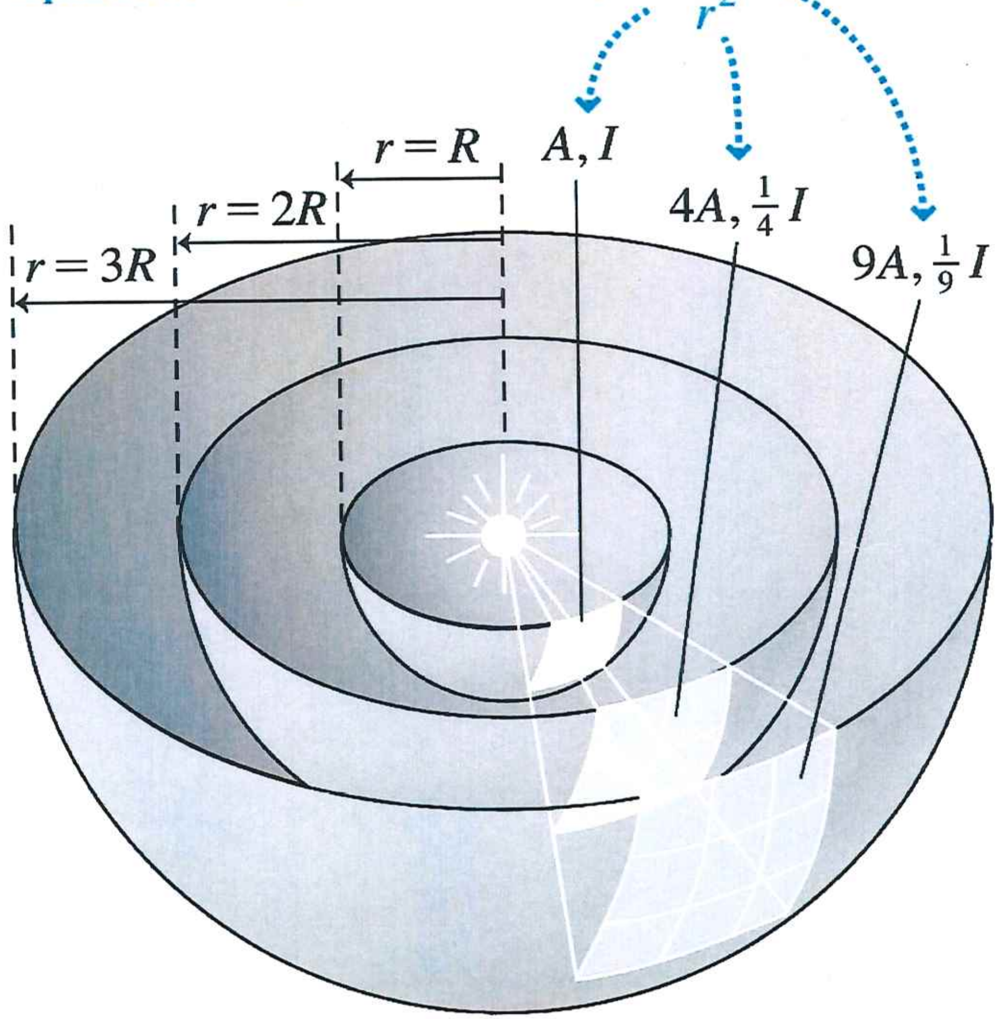
if $f\uparrow$ so $v\uparrow$

Sound Intensity

Figure 11.13

At a distance $r = R$, intensity $I = \frac{P}{A} = \frac{P}{4\pi r^2}$. $\left[\frac{W}{m^2}\right]$

As distance r increases, area A increases as its square: $A = 4\pi r^2$. Therefore, $I \propto \frac{1}{r^2}$.



$A = 4\pi R^2$

$A_2 = 4\pi(2R)^2 = 4\pi R^2 \cdot 4$

$A_3 = 4\pi(3R)^2 = 4\pi R^2 \cdot 9$

16. BIO The area of a typical eardrum is about 5.0×10^{-5} m^2. Calculate the sound power (the energy per second) incident on an eardrum at

a. the threshold of hearing and

b. the threshold of pain.

14.16 The sound power incident on the eardrum is $P = IA$, where I is the intensity of the sound and $A = 5.0 \times 10^{-5}$ m^2 is the area of the eardrum.

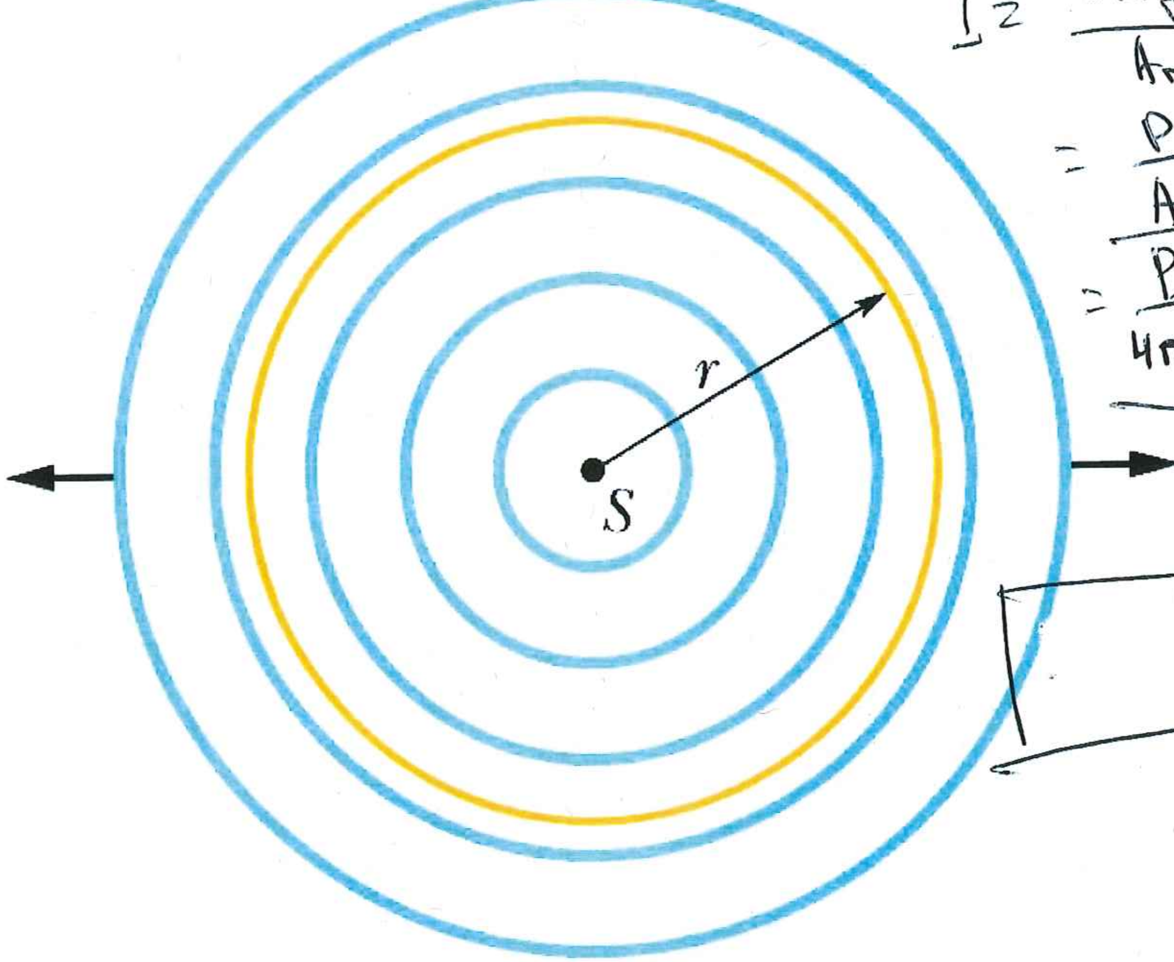
(a) At the threshold of hearing, $I = 1.0 \times 10^{-12}$ W/m^2, and

$$P = (1.0 \times 10^{-12} \text{ W/m}^2)(5.0 \times 10^{-5} \text{ m}^2) = \boxed{5.0 \times 10^{-17} \text{ W}}$$

(b) At the threshold of pain, $I = 1.0$ W/m^2, and

$$P = (1.0 \text{ W/m}^2)(5.0 \times 10^{-5} \text{ m}^2) = \boxed{5.0 \times 10^{-5} \text{ W}}$$

Intensity and Sound Level



$$I = \frac{\text{Energy Rate}}{\text{Area}} = \frac{P}{A} = \frac{P_{source}}{4\pi r^2} \rightarrow \frac{W}{m^2}$$

Reference Level! The decibel scale | human ear

since $I \sim 10^{-12}\,\frac{W}{m^2} \leftrightarrow 1\,\frac{W}{m^2}$

so the range we hear $\sim 10^{12}$

$$y = \log(10x) = \log 10 + \log x = 1 + \log x$$

Sound level $\beta = (10\text{ dB}) \log \frac{I}{I_0}$ | Alexander Graham Bell

where $I_0 = 10^{-12}\,W/m^2$

$I \rightarrow 10I$

$\beta \rightarrow \beta + 10$ (dB)

Conversation = 60 dB
Rock concert = 110 dB
Jet engine = 130 dB

Intensity Level in Decibels

threshhold of hearing: $\beta = 10\log\left(\dfrac{1.0\times10^{-12}\ \text{W/m}^2}{1.0\times10^{-12}\ \text{W/m}^2}\right) = 10\log(1) = 0\ \text{dB}$

$$\beta = 10\log\left(\frac{1.0\times10^{-11}\ \text{W/m}^2}{1.0\times10^{-12}\ \text{W/m}^2}\right) = 10\log(10) = 10\ \text{dB}$$

$$\beta = 10\log\left(\frac{1.0\times10^{-10}\ \text{W/m}^2}{1.0\times10^{-12}\ \text{W/m}^2}\right) = 10\log(100) = 20\ \text{dB}$$

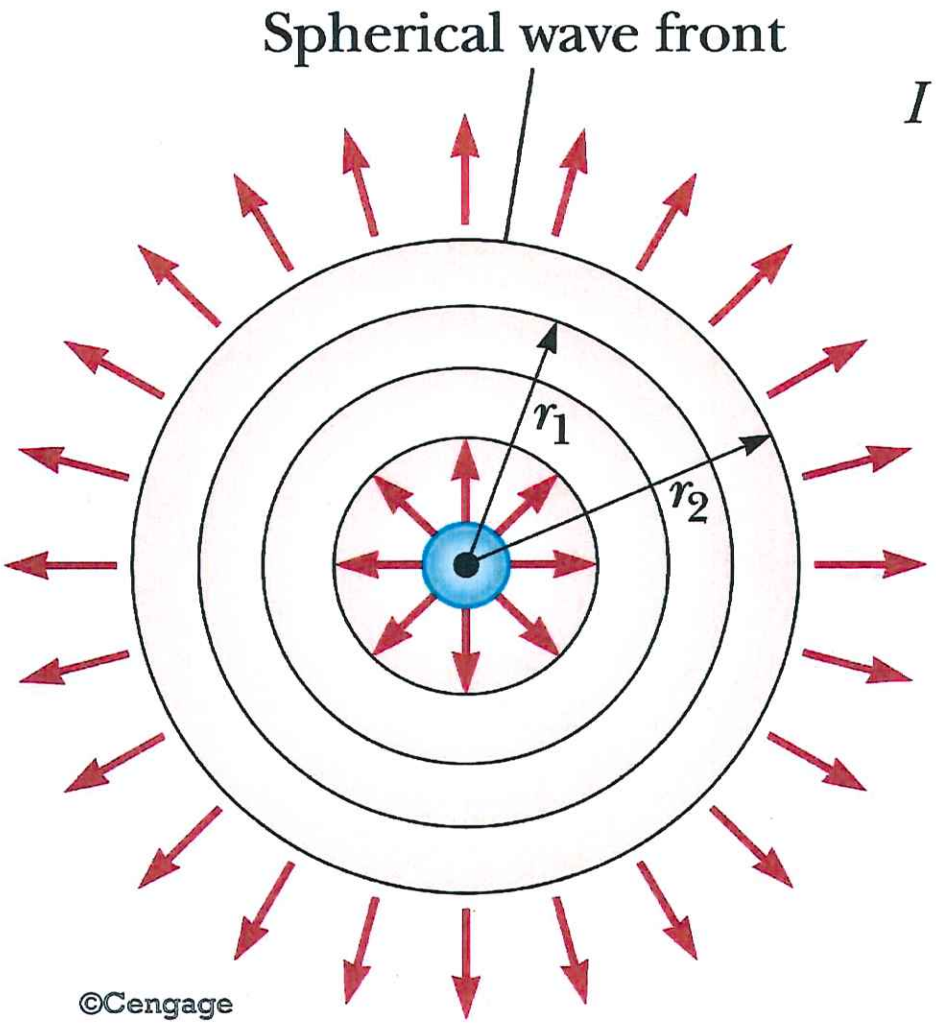
threshhold of pain: $\beta = 10\log\left(\dfrac{1}{1.0\times10^{-12}}\right) = 10\log\left(10^{12}\right) = 120\ \text{dB}$

What is the intensity level in decibels of a sound wave whose intensity is 10^{-6} W/m^2 ?

By definition β (dB) = 10 log (I/I_o) where $I_o = 10^{-12}$ W/m^2 and $I = 10^{-6}$ W/m^2

So β (dB) = 10 log (10^{-6} / 10^{-12}) = 10 log (10^{+6}) = 10 . 6 = 60 dB

Spherical and Plane Waves



$$I = \frac{\text{average power}}{\text{area}} = \frac{P_{av}}{A} = \frac{P_{av}}{4\pi r^2}$$

$$I_1 = \frac{P_{av}}{4\pi r_1^2} \quad I_2 = \frac{P_{av}}{4\pi r_2^2}$$

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

Chapter 14: Sound
Sound Waves – Sound Intensity

Example:

At the location of the Earth's upper atmosphere, the intensity of the Sun's light is 1400 W/m². What is the intensity of the Sun's light at the orbit of the planet Mercury?

$R_{ES} = 1.50 \times 10^{11}$ m

$R_{MS} = 5.85 \times 10^{10}$ m

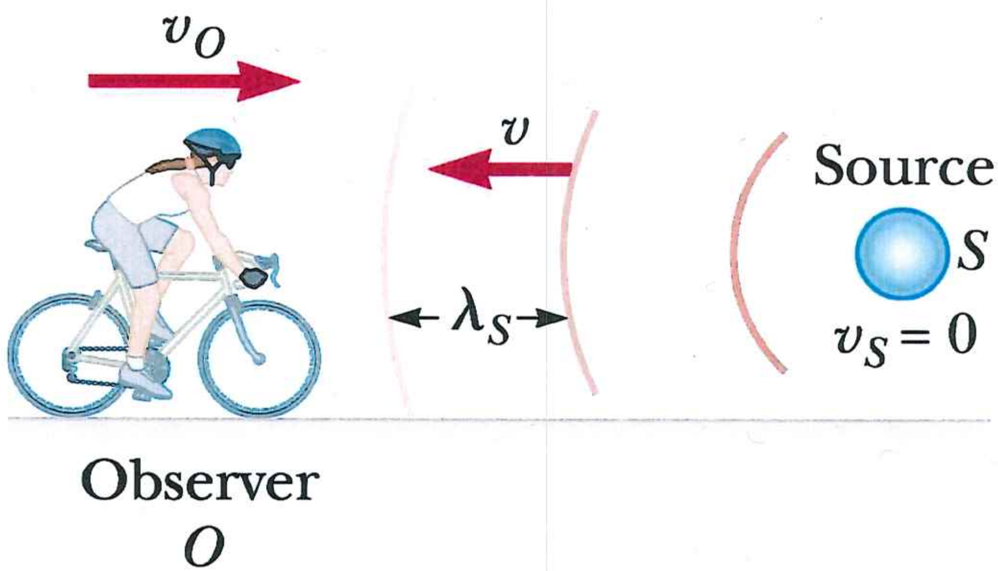
$$I_e = \frac{P_{sun}}{4\pi r_{es}^2} \qquad I_m = \frac{P_{sun}}{4\pi r_{ms}^2}$$

Divide one equation by the other:

$$\frac{I_m}{I_e} = \frac{\frac{P_{sun}}{4\pi\ r_{ms}^2}}{\frac{P_{sun}}{4\pi\ r_{es}^2}} = \left(\frac{r_{es}}{r_{ms}}\right)^2 = \left(\frac{1.50\times10^{11}\ \text{m}}{5.85\times10^{10}\ \text{m}}\right)^2 = 6.57$$

$$I_m = 6.57\, I_e = 9200\ \text{W/m}^2$$

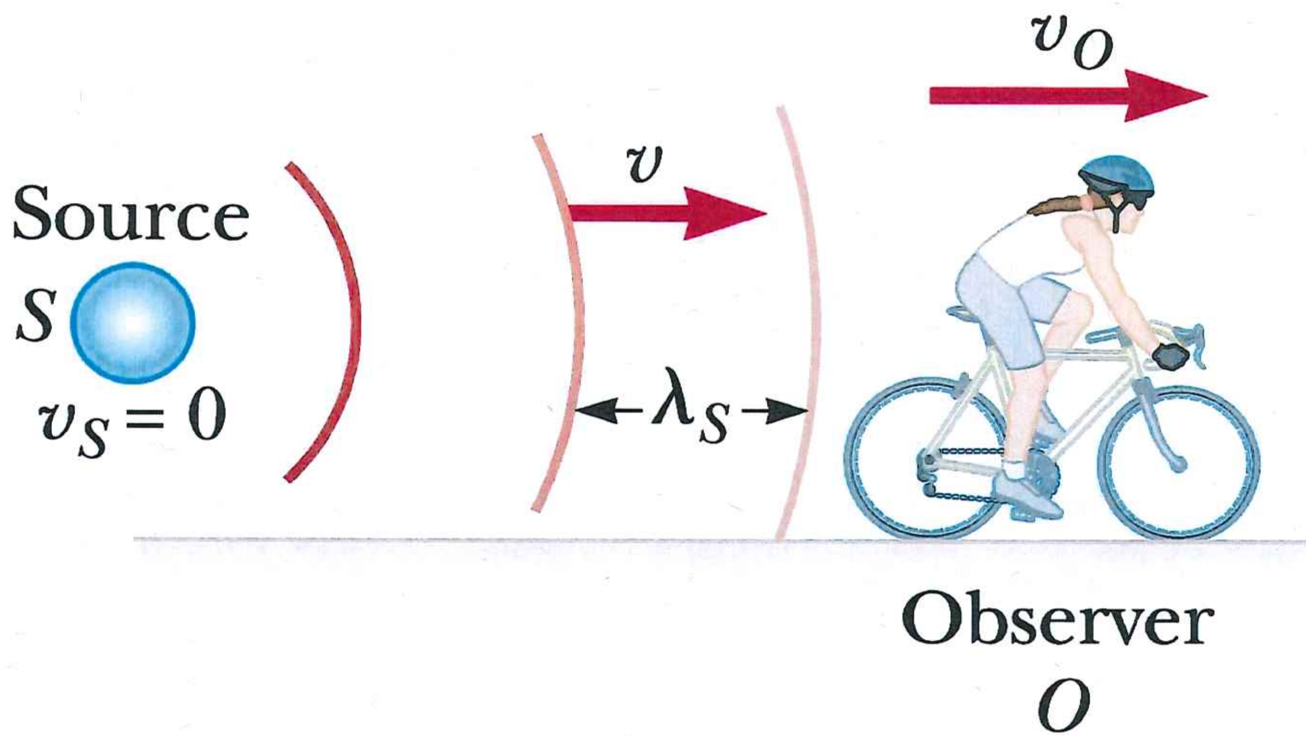
Case 1: The Observer Is Moving Relative to a Stationary Source



Additional wave fronts detected $= \dfrac{v_0 t}{\lambda_s}$ $\qquad$ $f_o = f_s + \dfrac{v_o}{\lambda_s}$

$\lambda_S = \dfrac{v}{f_S} \rightarrow f_O = f_S\left(\dfrac{v + v_0}{v}\right)$ (observer moving *toward* source)

Case 1: The Observer Is Moving Relative to a Stationary Source



$$f_O = f_S\left(\frac{v - v_0}{v}\right) \text{ (observer moving } \textit{away from} \text{ source)}$$

General Case

$$f_O = f_S\left(\frac{v + v_O}{v - v_S}\right)$$

25. A baseball hits a car, breaking its window and triggering its alarm which sounds at a frequency of 1 250 Hz. What frequency is heard by a boy on a bicycle riding away from the car at 6.50 m/s?

14.25 Here the sound source is stationary and the observer is moving away at 6.50 m/s so that $v_O = -6.50$ m/s and $v_S = 0$. The boy will hear a frequency lower than f_0. Take the speed of sound to be $v = 343$ m/s and substitute values into the Doppler shift equation:

$$f_O = f_S\left(\frac{v + v_O}{v - v_S}\right) = 1250\text{ Hz}\left(\frac{343\text{ m/s} - 6.50\text{ m/s}}{343\text{ m/s}}\right) = \boxed{1.23\times 10^3\text{ Hz}}$$

29. Two trains on separate tracks move toward each other. Train 1 has a speed of 1.30×10^2 km/h; train 2, a speed of 90.0 km/h. Train 2 blows its horn, emitting a frequency of 5.00×10^2 Hz. What is the frequency heard by the engineer on train 1?

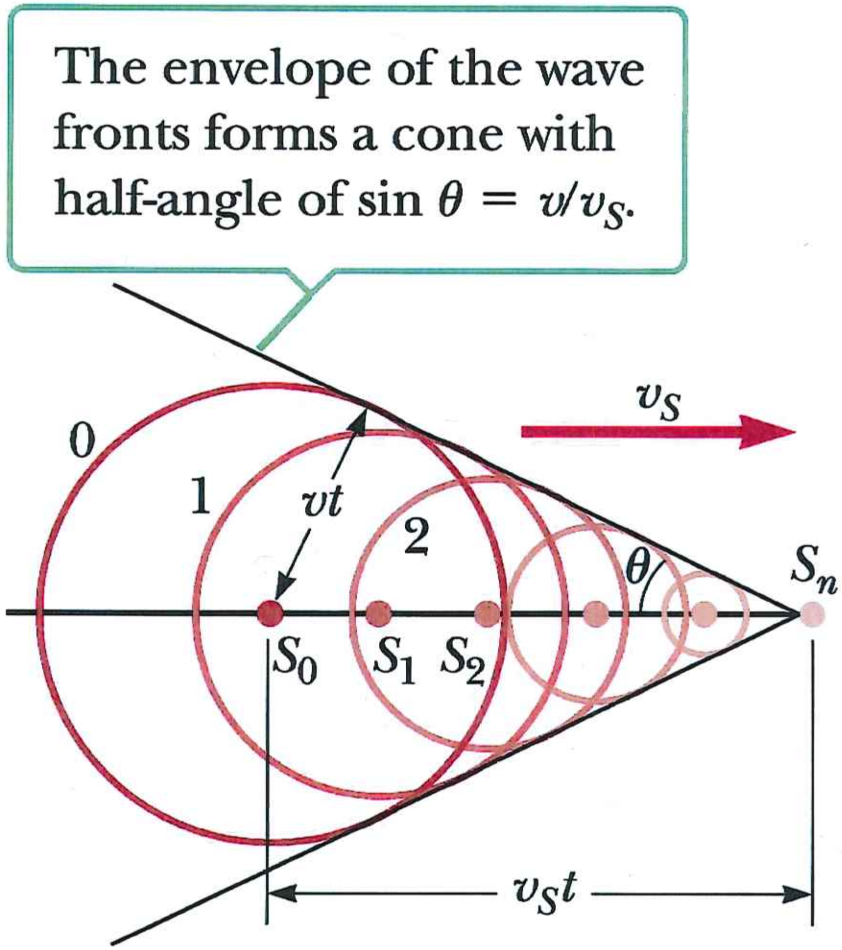
14.29 Both source and observer are in motion, so $f_O = f_S\left(\frac{v + v_O}{v - v_S}\right)$. Since each train moves *toward* the other, $v_O > 0$ and $v_S > 0$. The speed of the source (train 2) is

$$v_S = 90.0\frac{\text{km}}{\text{h}}\left(\frac{1\,000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 25.0\text{ m/s}$$

and that of the observer (train 1) is $v_O = 130$ km/h $= 36.1$ m/s. Thus, the observed frequency is

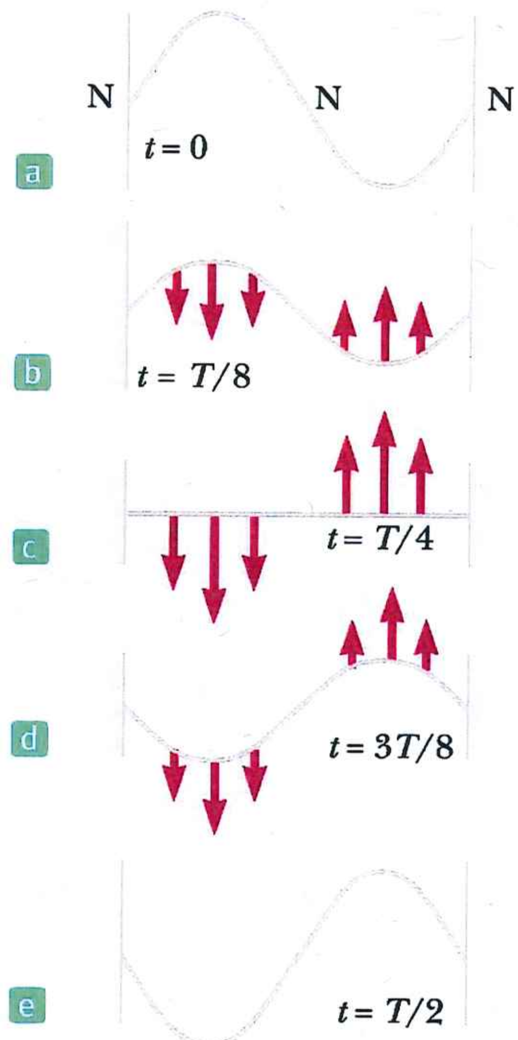
$$f_O = (500\text{ Hz})\left(\frac{343\text{ m/s} + 36.1\text{ m/s}}{343\text{ m/s} - 25.0\text{ m/s}}\right) = \boxed{596\text{ Hz}}$$

Shock Waves

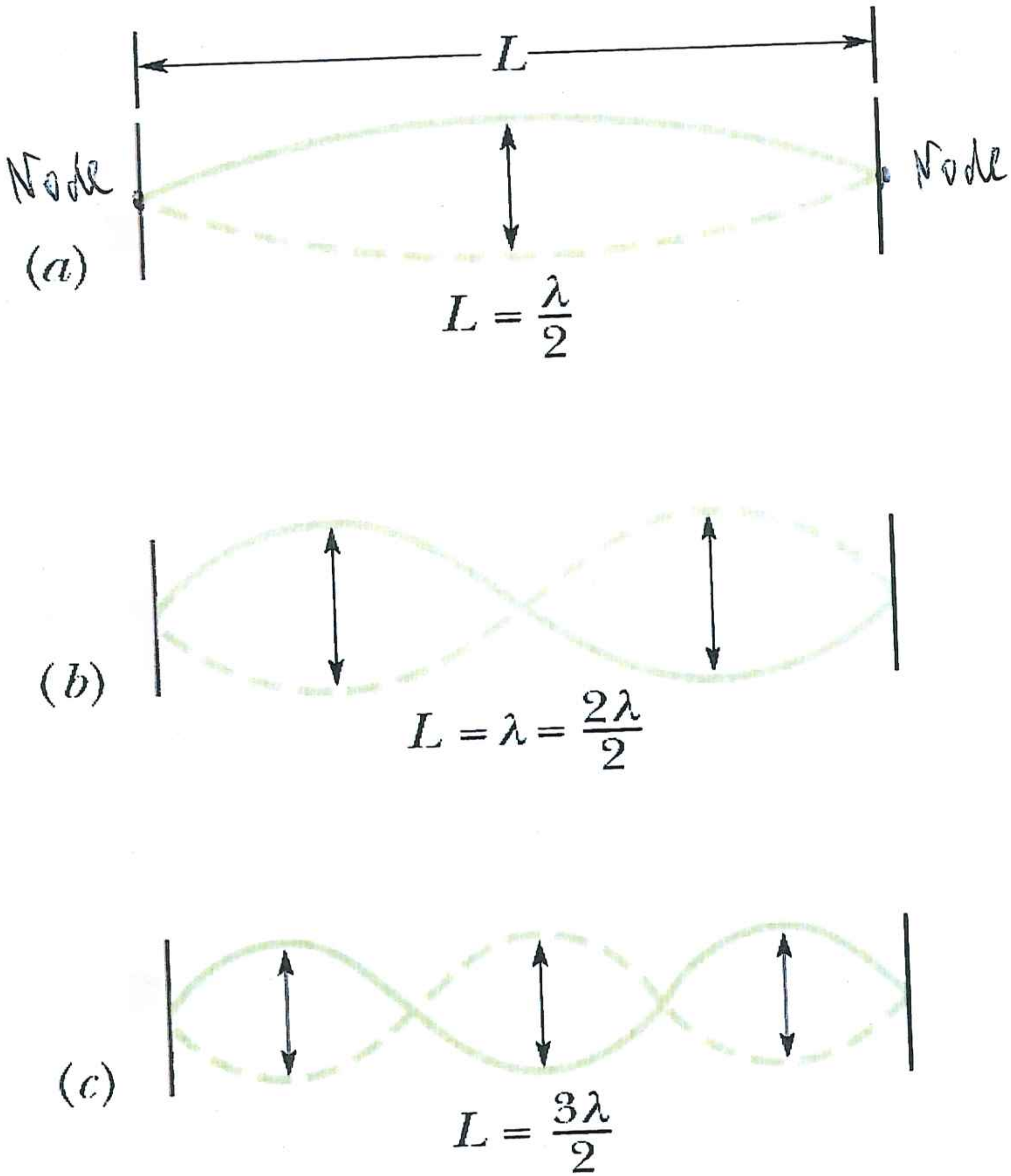


$$\sin\theta = \frac{v}{v_s}$$

Standing Waves



$$d_{NN} = \frac{1}{2}\lambda$$



Standing wave on a string of length L.

$$\lambda = \frac{2L}{n}, \quad n = 1, 2, 3$$

→ resonant frequencies

$$f = \frac{v}{\lambda} = n\frac{v}{2L}$$

Lowest resonant $f_1 = \frac{v}{2L}$ (fundamental mode)

(second harmonic) $f_2 = 2 \cdot \frac{v}{2L}$

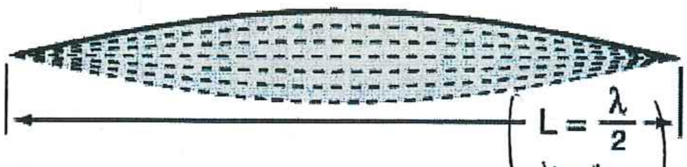
(third harmonic) $f_3 = 3 \cdot \frac{v}{2L}$

$f_1, f_2, f_3, \ldots$ harmonic series

8a

Vibrating String

The fundamental vibrational mode of a stretched string is such that the wavelength is twice the length of the string.



$\frac{V}{f} = \lambda = \frac{2L}{n}$; ($\lambda = 2L$, $n=1$)

$n = 1, 2, 3, 4$

Applying the basic wave relationship gives an expression for the fundamental frequency:

$V = \lambda \cdot f$

$$f_1 = \frac{v_{wave\ on\ string}}{2L}$$ **Calculation**

$f = n \frac{V}{2L}$

Since the wave velocity is given by $v = \sqrt{\frac{T}{m/L}}$, the frequency expression can be put in the form:

μ [kg/m]

n=1

$$f_1 = \frac{\sqrt{\frac{T}{m/L}}}{2L}$$

T = string tension
m = string mass
L = string length

$2f, 3f, 4f, \ldots$

The string will also vibrate at all harmonics of the fundamental. Each of these harmonics will form a standing wave on the string.

String frequencies	String instruments	Illustration with a slinky	Mathematical form

Index
Periodic motion concepts
Resonance concepts
HyperPhysics***** Sound
R Nave
Go Back

2:1 frequency → octave

1:5

42. A steel wire in a piano has a length of 0.700 0 m and a mass of 4.300×10^{-3} kg. To what tension must this wire be stretched so that the fundamental vibration corresponds to middle C ($f_C = 261.6$ Hz on the chromatic musical scale)?

14.42 In the fundamental mode of vibration, the wavelength of waves in the wire is

$$\lambda = 2L = 2(0.700\ 0\text{ m}) = 1.400\text{ m}$$

If the wire is to vibrate at $f = 261.6$ Hz, the speed of the waves must be

$$v = \lambda f = (1.400\text{ m})(261.6\text{ Hz}) = 366.2\text{ m/s}$$

The mass per unit length of the wire is

$$\mu = \frac{m}{L} = \frac{4.300\times10^{-3}\text{ kg}}{0.700\ 0\text{ m}} = 6.143\times10^{-3}\text{ kg/m}$$

and the required tension is given by $v = \sqrt{F/\mu}$ as

$$F = v^2\mu = (366.2\text{ m/s})^2(6.143\times10^{-3}\text{ kg/m} = \boxed{823.8\text{ N}}$$

51. BIO A 60.00-cm guitar string under a tension of 50.000 N has a mass per unit length of 0.100 00 g/cm. What is the highest resonant frequency that can be heard by a person capable of hearing frequencies up to 20 000 Hz?

14.51 The speed of transverse waves in the string is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{50.000\text{ N}}{1.000\,0 \times 10^{-2}\text{ kg/m}}} = 70.711\text{ m/s}$$

The fundamental wavelength is $\lambda_1 = 2L = 1.200\,0$ m and its frequency

is

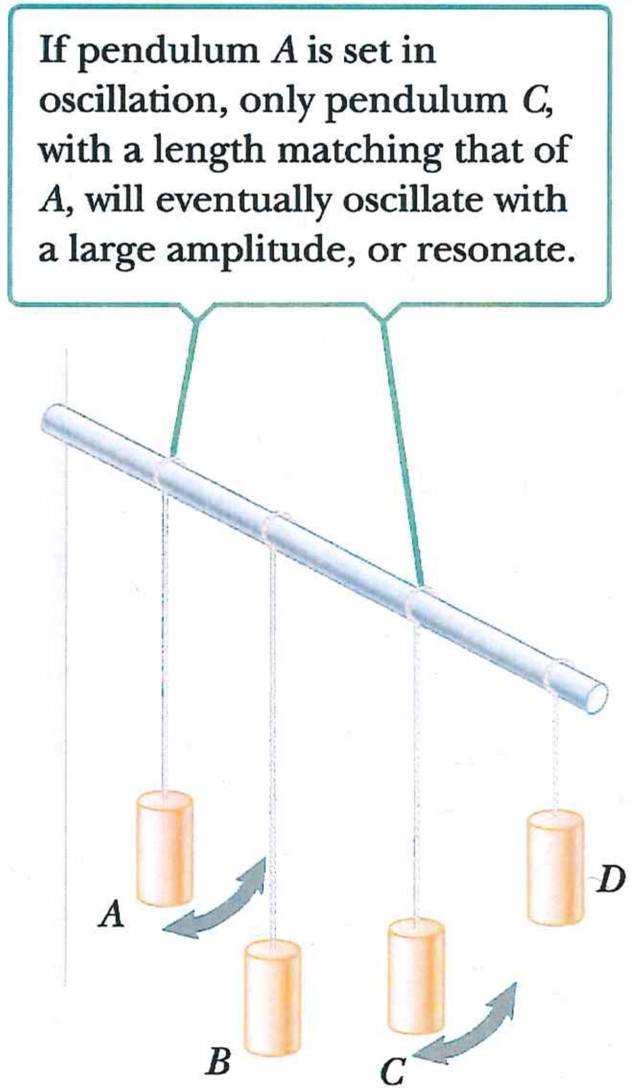
$$f_1 = \frac{v}{\lambda_1} = \frac{70.711\text{ m/s}}{1.200\text{ 0 m}} = 58.926\text{ Hz}$$

The harmonic frequencies are then

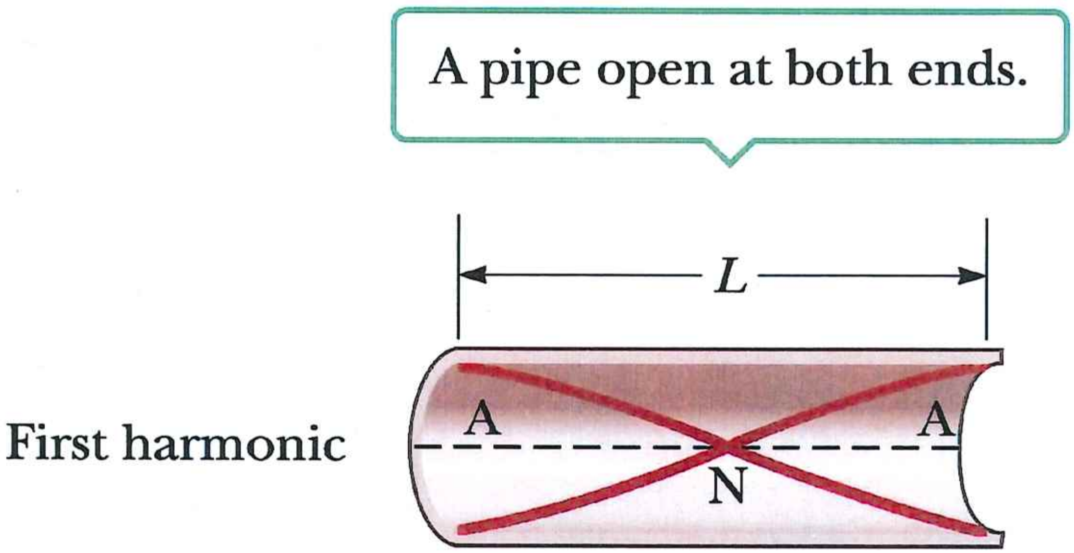
$f_n = nf_1 = n(58.926\text{ Hz})$, with n being an integer

The largest one under 20 000 Hz is $f_{339} = 19\,976\text{ Hz} = \boxed{19.976\text{ kHz}}$.

Forced Vibrations and Resonance



Standing Waves in Air Columns



$\lambda_1 = 2L$

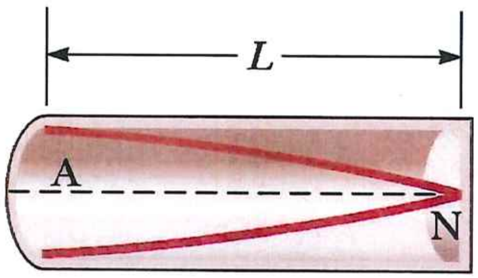
$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$

$$L = 2\left(\frac{\lambda_1}{4}\right) = \frac{\lambda_1}{2}$$

$$f = \frac{v}{\lambda_1} = \frac{v}{2L}$$

Standing Waves in Air Columns

A pipe closed at one end.

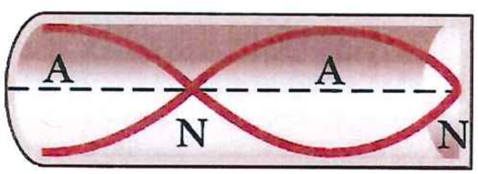


First harmonic

$$\lambda_1 = 4L$$

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$$

$$L = \frac{\lambda_1}{4} \rightarrow \lambda_1 = 4L$$



Third harmonic

$$\lambda_3 = \frac{4}{3}L$$

$$f_3 = \frac{3v}{4L} = 3f_1$$

$$L = 3\left(\frac{\lambda_3}{4}\right) \rightarrow \lambda_3 = \frac{4L}{3}$$

$$f_n = n\frac{v}{4L} = nf_1 \quad n = 1,\ 3,\ 5,\ \ldots$$

54. A pipe has a length of 0.750 m and is open at both ends.

a. Calculate the two lowest harmonics of the pipe.

b. Calculate the two lowest harmonics after one end of the pipe is closed.

14.54 **(a)** The harmonic frequencies of a pipe open at both ends are given by

$$f_n = nf_1 = n\frac{v}{2L} \qquad n = 1, 2, 3, \ldots$$

where v = 343 m/s is the sound speed and L = 0.750 m is the pipe's length. Substitute values to find

$$f_1 = (1)\frac{v}{2L} = \frac{343 \text{ m/s}}{2(0.750 \text{ m})} = \boxed{229 \text{ Hz}}$$

$$f_2 = 2f_1 = \boxed{457 \text{ Hz}}$$

(b) The harmonic frequencies of a pipe closed at one end are given by

$$f_n = nf_1 = n\frac{v}{4L} \qquad n = 1, 3, 5, \ldots$$

Substitute values to find

$$f_1 = (1)\frac{v}{4L} = \frac{343 \text{ m/s}}{4(0.750 \text{ m})} = \boxed{114 \text{ Hz}}$$

$$f_3 = 3f_1 = \boxed{343 \text{ Hz}}$$

59. A pipe open at both ends has a fundamental frequency of 3.00×10^2 Hz when the temperature is 0° C.

a. What is the length of the pipe?

Answer

b. What is the fundamental frequency at a temperature of 30.0° C?

14.59 **(a)** The fundamental wavelength of the pipe open at both ends is $\lambda_1 = 2L = v/f_1$. Since the speed of sound is 331 m/s at 0°C, the length of the pipe is

$$L = \frac{v}{2f_1} = \frac{331\text{ m/s}}{2(300\text{ Hz})} = \boxed{0.552\text{ m}}$$

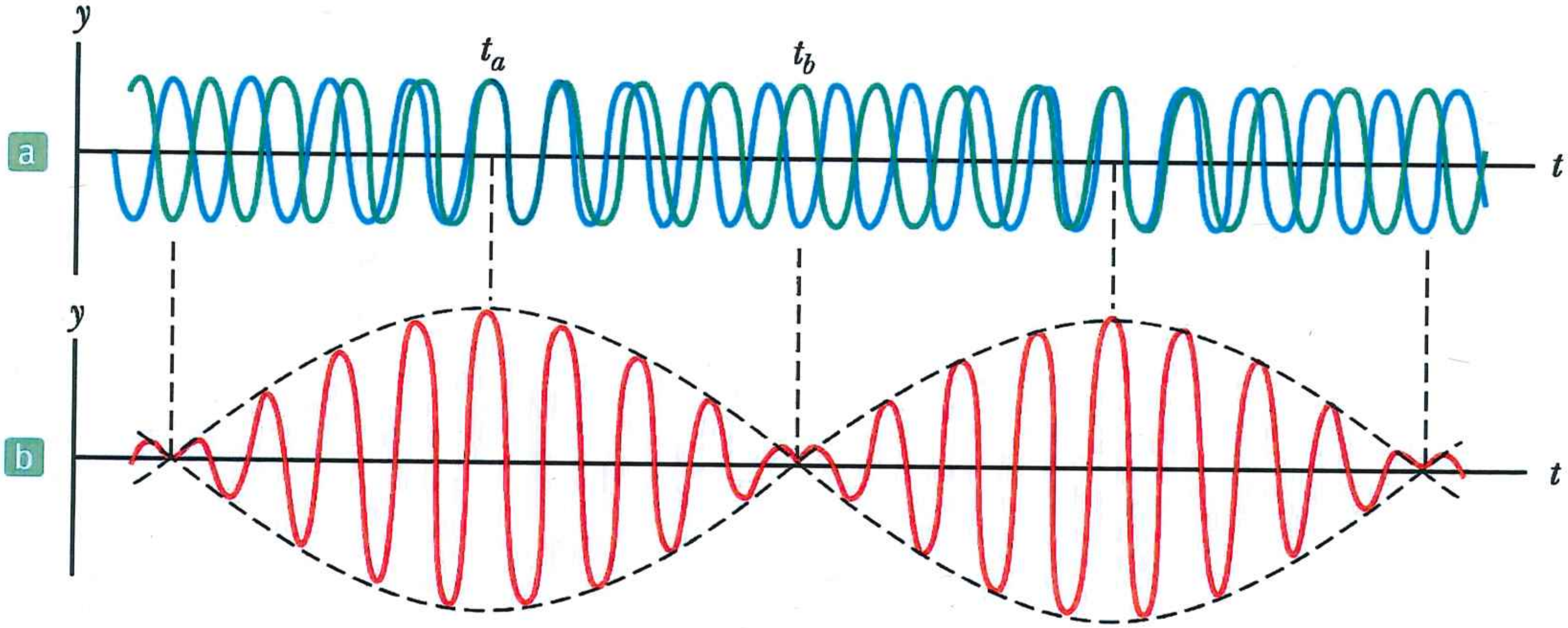
(b) At $T = 30°\text{C} = 303$ K,

$$v = (331\text{ m/s})\sqrt{\frac{T_K}{273}} = (331\text{ m/s})\sqrt{\frac{303}{273}} = 349\text{ m/s}$$

and

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L} = \frac{349\text{ m/s}}{2(0.552\text{ m})} = \boxed{316\text{ Hz}}$$

Beats



$$f_b = |f_2 - f_1|$$

61. A guitarist sounds a tuner at 196 Hz while his guitar sounds a frequency of 199 Hz. Find the beat frequency.

14.61 The beat frequency is $f_b = |f_2 - f_1| = |196\text{ Hz} - 199\text{ Hz}| = \boxed{3\text{ Hz}}$.